Ques. 1(a): What are those revisions done to the modern nebular hypothesis on the evolution of the solar system? (b) Why was Urey's hypothesis of the evolution of the Moon rejected? (c) Pluto is not a planet. Write two points in favor of this statement.

Ans 1(a): Molecular gas and dust began to collapse gravitationally at the centre of a giant cloud into denser regions. As the denser regions pulled in more and more matter, conservation of momentum caused it to begin rotating faster and faster, while increasing pressure caused it to heat up. Most of the materials ended up in a ball at the centre while the rest of the matter flattened out into disk that circled around it. While the ball at the centre formed the Sun, the rest of the material would form into the protoplanetary disc.

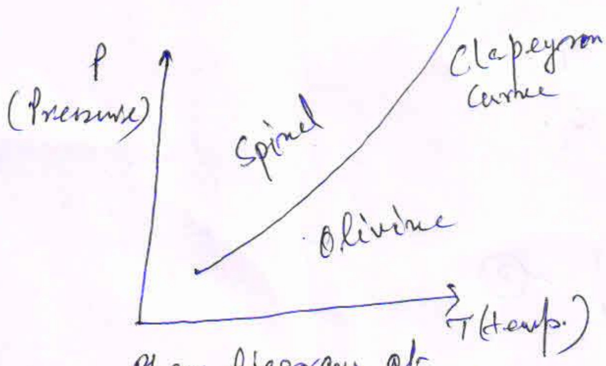
Distribution of angular momentum between planets and Sun. More than 99.9% of the total mass of the solar system is concentrated in the Sun, however more than 99% of the angular momentum is carried by the orbital motion of the planets, especially the four great planets. Of these, Jupiter is a special case: it accounts for over 70% of the mass and more than 60% of the angular momentum of the planets. It is thought that at some point an increase in the intensity of the solar wind cleared the remaining gas and dust out of the system, and the solution to angular momentum problem seems to lie in this solar wind. The loss of mass via stellar wind was sufficient to reduce the rate of the Sun's rotation. Thus, the planets preserved the angular momentum that was in the original solar nebula, but the Sun has gradually slowed down since 4.53 Ga before.

Ans 1(b): Deficiency of siderophile elements in the lunar crust and low density of the Moon compared to the Earth could not be explained by Urey's hypothesis. Additionally, compared with Earth, lunar rocks were also discovered to be more depleted in so-called volatile elements — those that vaporize easily upon heating — a hint that the Moon formed at high-temperatures.

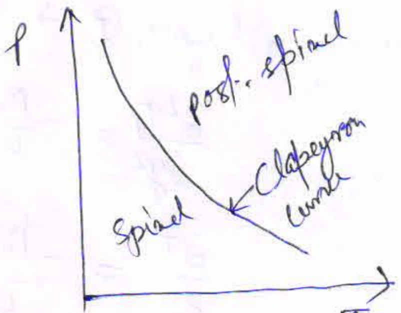
Ans 1(c): Pluto being a "dwarf planet", has no enough mass, and shares its orbital neighborhood with Kuiper belt objects such as the plutinos.

Ques. 2: Explain through a phase diagram the reasons for the elevation of the olivine-spinel phase boundary and depression of the *spinel–perovskite* phase *boundary* in a subducting oceanic lithosphere.

Ans 2:



Phase diagram at 400 km boundary



Phase diagram at 670 km boundary.

The olivine-spinel boundary is elevated in the descending oceanic lithosphere as compared with its position in the surrounding mantle because the pressure at which the phase change occurs depends on temperature. The left sketch is a plot of the Clapeyron curve, which gives the pressures and temperatures at which two phases of the same material, such as olivine and spinel, are in equilibrium. The slope of the Clapeyron curve γ is given by

$$\gamma = \frac{dp}{dT}$$

$$or, \frac{dz}{dT} = \frac{\gamma}{\rho G}$$

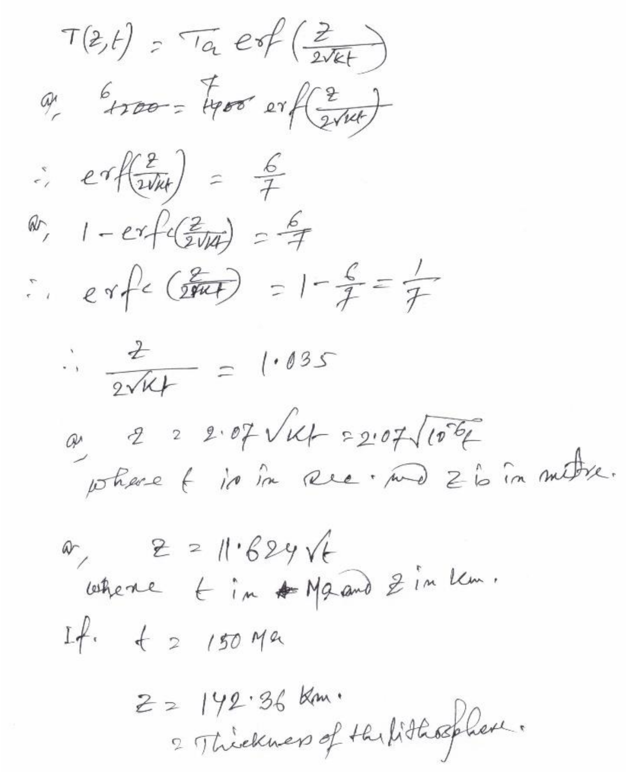
where dp = gρdz.

For the olivine to spinel phase change at 400 km depth, the slope of the Clapeyron curve is positive. Since dT is negative for the lower temperatures in the interior of the descending lithosphere, and the olivine-spinel phase change occurs at a shallower depth in the slab. Therefore, the olivine-spinel boundary is elevated inside the descending lithosphere. The elevation of this phase change boundary inside the descending lithosphere increases in density associated, and thereby adding more forces driving the plate downward into the mantle.

Instead, for spinel–perovskite phase boundary (right plot) at 670 km depth, the slope of the Clapeyron curve is negative. Since dT is negative for the lower temperatures in the interior of the descending lithosphere, and the spinel–perovskite phase change occurs at a deeper level in the slab. Therefore, the spinel– perovskite phase boundary is depressed inside the descending lithosphere. The mass deficiency happens inside the descending lithosphere because of the depression of this phase boundary, and the top of the lithosphere is impeded, and suffers back-thrust, which is responsible for the generation of deep focus earthquake. This also opposes the migration of the lower mantle materials into the upper mantle, and allows layered-earth convection.

Ques. 3: Derive the relationship between the oceanic lithospheric thickness and its age of evolution. Take the asthenospheric temperature as 1400°C. The temperature at the base of the lithosphere is 1200°C. $T(z,t) = T_a \operatorname{erf}\left(\frac{z}{2\sqrt{kt}}\right)$ T is temperature, z is depth, t is lithospheric age and k is thermal diffusivity of the oceanic lithosphere. Use the supplied error function table for the computation. Hence, calculate the thickness of the oceanic lithosphere near the Java region of 150 Ma age. Take k = 10^{-6} m^2/sec

Ans. 3:



$$T(z,t) = T_a \operatorname{erf}\left(\frac{z}{2\sqrt{kt}}\right)$$

or, $\overset{6}{\cancel{1200}} = \overset{7}{\cancel{1400}} \operatorname{erf}\left(\frac{z}{2\sqrt{kt}}\right)$

$\therefore \operatorname{erf}\left(\frac{z}{2\sqrt{kt}}\right) = \frac{6}{7}$

or, $1 - \operatorname{erfc}\left(\frac{z}{2\sqrt{kt}}\right) = \frac{6}{7}$

$\therefore \operatorname{erfc}\left(\frac{z}{2\sqrt{kt}}\right) = 1 - \frac{6}{7} = \frac{1}{7}$

$\therefore \frac{z}{2\sqrt{kt}} = 1.035$

or, $z = 2.07\sqrt{kt} = 2.07\sqrt{10^{-6}t}$

where t is in sec. and z is in metre.

or, $z = 11.624\sqrt{t}$

where t in Ma and z in km.

If. $t = 150$ Ma

$z = 142.36$ km.

= Thickness of the lithosphere.

Ques. 4: Write two points each in favor of (a) whole mantle convection, (b) layered mantle convection. ~~(c)~~ How do you explain that the turbulence is negligible in the mantle convection process?

Ans 4(a): Many huge slabs of ocean crust that have been dragged down, or subducted, into the mantle can still be detected in the deep Earth. These slabs slowly sink downward toward the bottom of the mantle. Some slabs sink all the way down, providing evidence for global stirring of the mantle by a process called "whole-mantle convection." However, a large number of these slabs have stalled out and appear to float 1,000 kilometers deep, indicating a notable change in physical properties with depth.

Ans 4(b): A two-layer model of the convection pattern inside the Earth can be inferred from the surface expressions, i.e., the long-lived hot spots13 and the subducting plates. Most of the surface complexity comes from upper mantle and TZ phenomena.

seismic tomography—creating images of Earth's interior using earthquake-generated waves—provides evidence that the whole mantle is stirred, and presumably well-mixed.

Ans 4(c): The Reynolds number is found to be 1.5×10^{-20} for the mantle, and is so small that turbulence is negligible. Similar results are also found by considering the upper or lower mantle alone. Clearly, although mantle convection involves high Rayleigh numbers (implying vigorous convection on a geological timescale), it takes place by laminar flow.

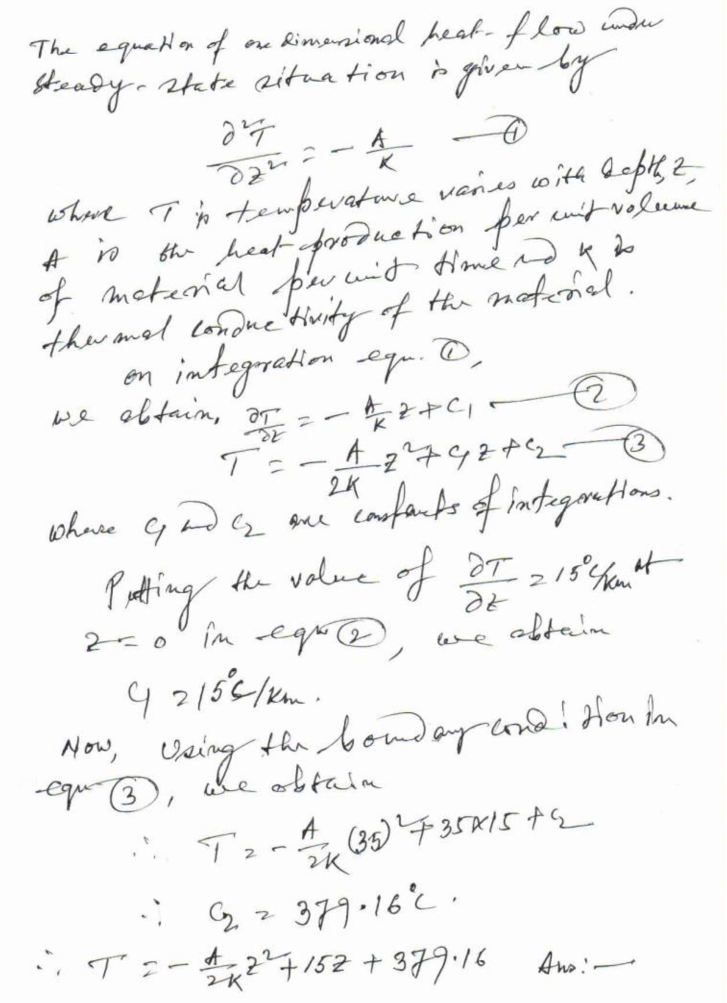
Ques. 5: Calculate an equilibrium geotherm from a one-dimensional heat-flow equation given the following boundary conditions:

a. $\frac{\partial T}{\partial z} = 30°\mathrm{C}\ km^{-1}$ at z = o-km and

b. T = 700°C at z = 35 km

Assume that the internal heat generation is 1.0 $\mu\mathrm{Wm}^{-3}$ and the thermal conductivity is 3 $\mathrm{Wm}^{-1}°\mathrm{C}^{-1}$. Take the steady-state one-dimensional heat conduction equation as $\frac{\partial^2 T}{\partial Z^2} = -\frac{A}{K}$, A is radioactive heat generation and K is thermal conductivity. Hence, calculate the temperature at a depth of 25 km.

Ans 5:



The equation of one dimensional heat-flow under steady-state situation is given by

$$\frac{\partial^2 T}{\partial z^2} = -\frac{A}{K} \longrightarrow ①$$

where T is temperature varies with depth, z, A is the heat-production per unit volume of material per unit time and K is thermal conductivity of the material.

on integration equ. ①,

we obtain, $\frac{\partial T}{\partial z} = -\frac{A}{K}z + C_1 \longleftarrow ②$

$$T = -\frac{A}{2K}z^2 + c_1 z + c_2 \longrightarrow ③$$

where c_1 and c_2 are constants of integrations.

Putting the value of $\frac{\partial T}{\partial z} = 15°C/km$ at $z = 0$ in eqn ②, we obtain

$c_1 = 15°C/km$.

Now, Using the boundary condition in eqn ③, we obtain

$$\therefore T = -\frac{A}{2K}(35)^2 + 35 \times 15 + c_2$$

$$\therefore c_2 = 379.16°C.$$

$\therefore T = -\frac{A}{2K}z^2 + 15z + 379.16$ Ans:—

Here, for z = 25 km,

$$T = -\frac{1}{2 \times 3} \times 25^2 + 15 \times 25 + 379.16 = 649.99℃$$

Therefore, the temperature at a depth of 25 km is 649.99°C

Ques.6(a): In a heat-flow province the heat production by the radioactive sources is $101 \mathrm{Wkg}^{-1}$. Assuming the radioactive heat flow is only due to a layer of thickness H escaping vertically Calculate the value of H. Assume the density of the layer to be $2650 \mathrm{kg/m}^3$, reduced heat flow and total heat flow of the province are $33 \mathrm{mWm}^{-2}$ and $86 \mathrm{mWm}^{-2}$, respectively.

(b) Derive the expression of adiabatic temperature gradient inside the earth, and hence b) calculate the adiabatic temperature gradient at a depth of 1700km inside the earth using: $g = 10.2$ m-s^{-2}, $c_p = 1250$ J kg^{-1} °K^{-1}, $\alpha = 9.8\times10^{-6}$ °K^{-1}, and T = 2500°K

Ans. 6(a):

We know,

$$q = q_0 + HA.$$

or, $86 = 33 + HA$

$\therefore$ $HA = 86 - 33 = 53$

$$\therefore H = \frac{53}{A}$$

$$= \frac{53 \text{ mW/m}^2}{101 \times 2650 \text{ W/m}^3}.$$

$$= \frac{53}{101 \times 2650 \times 10^3} \frac{\text{Km}}{10^3}.$$

$= 1.98 \times 10^{-10}$ Km.

= Thickness of the layer.

Ans. 6(b): We obtained from Maxwell's laws of thermodynamic:

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\text{or, } \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial S}\right)_P$$

where T is temperature, P is pressure, V is volume and S is entropy.

We know that the volume coefficient of expansion of a material is given by

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P.$$

If ΔQ amount of heat is required to raise the temperature by an amount ΔT, we can write

$\Delta Q = C_P m \Delta T$, where m is mass and C_P is specific heat of the material at constant pressure.

If the change in entropy of the system containing the material is ΔS, we can write

$$\Delta Q = T\Delta S, \text{and } \therefore \left(\frac{\partial T}{\partial S}\right)_P = \frac{T}{mC_P}$$

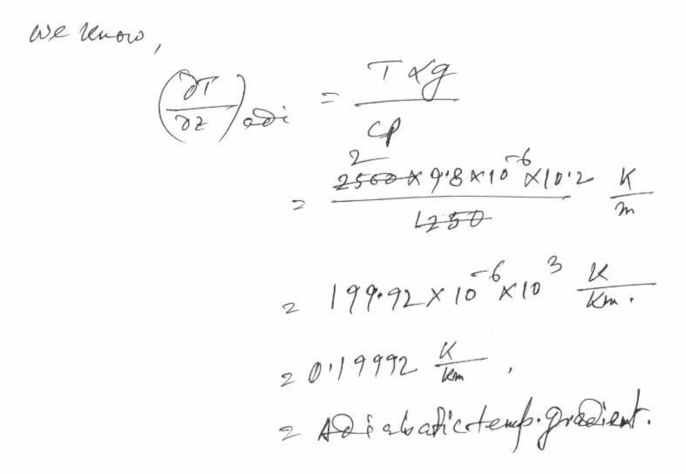
$$\therefore \left(\frac{\partial T}{\partial P}\right)_S = T\frac{\alpha V}{C_P\, m} = T\frac{\alpha}{\rho C_P}$$

Now, substituting dp = ρg dz in the above equation, we have

$$\therefore \left(\frac{\partial T}{\partial Z}\right)_{\text{adiabatic}} = T\frac{\alpha g}{C_P}$$

The left-hand side of the above equation represents the adiabatic change of temperature with depth inside the Earth.

Ans 6(c):



We know,

$$\left(\frac{\partial T}{\partial z}\right)_{adi} = \frac{T\alpha g}{C_p}$$

$$= \frac{\overset{2}{\cancel{2500}} \times 9.8\times 10^{-6} \times 10.2}{\cancel{1250}}\ \frac{K}{m}$$

$$= 199.92 \times 10^{-6} \times 10^{3}\ \frac{K}{Km.}$$

$$= 0.19992\ \frac{K}{Km},$$

= Adiabatic temp. gradient.